Department of Computer Science

A.Y. 2019/2020

LOGISTICS PROJECT REPORT

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**Problem**

*"Car Transportation s.p.a. must send, at minimum cost, 5 cars from node 1 and 6 cars from node 4. The destinations are nodes 7 and 8, with demand of 7 and 4 respectively. The capacity of the arches (4,5) and (6,7) is 7, while the capacity of each other arch is 4."*

*Figure 1: graphical representation of the network G = (V, A)*

**1. Solution to point 1**

The problem we want to analyze falls into the category of Minimum Cost Flow Problems. The goal is to determine how to send cars along the arcs, from two origins (node 1 and node 4) to two destinations (node 7 and node 8), considering the capacity constraint of each arc, in order to fully satisfy the demand and minimize the transportation costs.

**1.1 INPUT DATA**

Let G = (V, A) be the network in which the cars are transported. V defines the set of nodes, while A, the uni-directional set of arcs.

V = [1, 2, 3, 4, 5, 6, 7, 8]

A = [(1,4),(1,3),(1,2),(2,3),(2,6),(3,5),(3,6),(4,5),(5,6),(5,8),(6,8),(7,8)]

c{i,j} defines the transportation cost of each individual car from node i to node j, ∀(i,j) ∈ A, while u{i,j} defines the maximum capacity (upper capacity) that can pass form arc i to arc j, ∀(i,j) ∈ A

Arcs (A) (1,2) (1,3) (1,4) (2,6) (2,3) (3,6) (3,5) (4,5) (5,6) (5,8) (6,7) (7,8) Cost (ci,j) 2 3 7 8 5 2 4 4 1 2 9 2 Capacity (ui,j) 4 4 4 4 4 4 4 7 4 4 7 4

bi defines the balance of node *i*, ∀i ∈ V. Depending of the sign of bi, we can differentiate between:

● bi > 0: destination node, where the value of birepresents the total demand of cars for node i, in other words the quantity of cars that must remain in the node at the end of the transportation.

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● bthe i < quantity 0: source of node, goods where that must the value leave of that bi represents node and be the sent supply to other.

of cars for node i, in other words

● bwithout i = 0: transshipment node, in which all incoming goods are immediately leaving any goods in node i (for this reason the balance is zero).

sent to the next node,

Nodes (*V*) 1 2 3 4 5 6 7 8 Balance (bi) -5 0 0 -6 0 0 7 4

**1.2 DECISION VARIABLE AND OBJECTIVE FUNCTION**

The decision variable (*flow variable*), defined as xi,j, indicates the quantity of goods to be sent from arc *i* to arc *j*, ∀(i,j) ∈ A. The objective function is expressed as a minimization of the transportation cost of each individual vehicle from *i* to *j*, for the quantity of cars to be sent from *i* to *j*, ∀(i,j) ∈ A.

min ∑ cijxij

(i,j)∈A = min (2 ∗ x12 + 3 ∗ x13 + 7 ∗ x14 + 8 ∗ x26 + 5 ∗ x23 + 2 ∗ x36 + 4 ∗ x35 + 4 ∗ x45 + 1 ∗ x56 + 2 ∗ x58 + 9 ∗ x67 + 2 ∗ x78)

**1.3 CONSTRAINTS**

The first constraint we need to put is the *flow conservation constraint*. It is a constraint that, by setting the difference between sum of the set of xij belonging to the backward star and the sum of the set of xij belonging to the set of Forward star (the cars incoming and e leaving respectively), enables that the exact the node bi good remains in the node after the transportation. In other words, all good in excess can enter

but must leave *i*:

∑ xji (j,i)∈BS(i)

− ∑ xij (i,j)∈FS(i)

= bi,∀i ∈ N

−x12 − x13 − x14 = −5

x12 − x23 − x26 = 0

x13+x23 − x35 − x36 = 0

x14 − x45 = −6

x45 + x35 − x56−x58 = 0

x26 + x36 + x56 − x67 = 0

x67 − x78 = 7

x58 + x78 = 4

The second constraint we need to put is the mix between *capacity constraint* and *non-negativity constraint*. It permits that both the capacity of each arc is respected, and the quantity of each arc is positive:

0 ≤ xij ≤ uij,∀(i,j) ∈ A

0 ≤ x12 ≤ 4

0 ≤ x13 ≤ 4

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0 ≤ x14 ≤ 4

0 ≤ x23 ≤ 4

0 ≤ x26 ≤ 4

0 ≤ x35 ≤ 4

0 ≤ x36 ≤ 4

0 ≤ x45 ≤ 7

0 ≤ x56 ≤ 4

0 ≤ x58 ≤ 4

0 ≤ x67 ≤ 7

0 ≤ x78 ≤ 4

We have to introduce another constraint, in order to have a feasible solution. The sum of the balance of all nodes, must be 0.

∑bi i∈V

= 0,∀i ∈ V

In the end, since this problem a *Minimum Cost Flow Problem*, we must remember the *integrality property*. If the balances of the nodes bi are integer values ∀i ∈ V, and the capacity for every arch uij is integer, ∀(i,j) ∈ A, then there exists an optimal solution formed by integer values.

**2. Solution to point 2**

By running the problem wrote in AMPL language, the solution obtained, defined as the number of cars to pass through each node, is:

Arcs (A) (1,2) (1,3) (1,4) (2,6) (2,3) (3,6) (3,5) (4,5) (5,6) (5,8) (6,7) (7,8) Quantity (xij) 1 4 0 1 0 4 0 6 2 4 7 0

*Figure 2: Representation in the network of the optimal solution for point 1*

The total transportation costs, obtained as the multiplication of the quantity of goods that passes on every arc and its unitary cost, for every arc in the network, is 127.

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**3. Solution to point 3**

*“We assume* cij *to be the cost associated to every arc* (i,j)*, that represents the unitary cost for increasing the capacity of* (i,j)*, with respect to the input capacity, as previously specified. Formulate the problem to ship 30 cars from node 1 to node 8, minimizing the cost of increasing the capacity of the arc.”*

To solve the problem of point three, we change the solution to problem one into a problem which is similar to the *Shortest Path Problem*, for two reasons: (1) Since now, there is only one origin and one destination (node 1 and node 8 respectively), (2) the quantity to be sent and to be received is the same, so we can assume that we have to send one unit of good from the origin to the destination, with the goal to minimize the time of the transportation. The time in this case, is depicted by the cost of enlarging the capacity of the arcs in the network. However, to do this, we have to take into account the quantity to be shipped (30) into the computation, in order to have a correct estimation of the total enlargement cost.

**3.1 INPUT DATA**

From the previous problem, only few variables have changed. The network G = (V,A), the set of nodes V and the set of arcs A remains the same.

V = [1, 2, 3, 4, 5, 6, 7, 8]

A = [(1,4),(1,3),(1,2),(2,3),(2,6),(3,5),(3,6),(4,5),(5,6),(5,8),(6,8),(7,8)]

The transportation cost, cij, has become the unit cost for increasing the capacity of one node for one unit, and it is now included in a new variable introduced for the purpose of solving this problem. The maximum capacity for each arc, uij, is no more a constraint but becomes part of the objective function, as we will see in the further paragraph. To solve the problem, we introduce two new variables:

● D, which represent the quantity of cars to be shipped from the origin to the destination, in this case D = 30;

● aij, which represent the total cost for enlarging the capacity of an arc until D units can pass through it ∀(i,j) ∈ A. For every arc, it considers the lack of capacity given by the difference between the existing capacity, uij, and the total demand, D. To have the total cost we multiply the lacking capacity by the unit cost for enlarging the arc, cij:

aij = cij ∗ (D − uij),∀(i,j) ∈ A

a12 = 2 ∗ (30 − 4) = 52

a13 = 3 ∗ (30 − 4) = 78

a14 = 7 ∗ (30 − 4) = 182

a23 = 5 ∗ (30 − 4) = 130

a26 = 8 ∗ (30 − 4) = 208

a35 = 4 ∗ (30 − 4) = 104

a36 = 2 ∗ (30 − 4) = 52

a45 = 4 ∗ (30 − 7) = 92

a56 = 1 ∗ (30 − 4) = 26

a58 = 2 ∗ (30 − 4) = 52

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a67 = 9 ∗ (30 − 7) = 207

a78 = 2 ∗ (30 − 4) = 52

With this new constraint we can redraw the network showing for each arc the total enlargement cost, as we can see from the table and the graph below.

Arcs (*A*) (1,2) (1,3) (1,4) (2,6) (2,3) (3,6) (3,5) (4,5) (5,6) (5,8) (6,7) (7,8) Enlarging Cost (aij) 52 78 182 208 130 52 104 92 26 52 207 52

*Figure 3: Representation of the total enlargement cost for every arc in the network*

The for the balance destination variable node, bi, since we included and 0 for all the total demand in D, takes value -1 in for the origin node, 1

other nodes.

**3.2 DECISION VARIABLE AND OBJECTIVE FUNCTION**

The decision variable, xij, since we included the total demand of cars in D, has turned into a binary variable, with value 1 if the arc is selected for the enlargement and 0 otherwise, ∀(i,j) ∈ A. In other words, if we set xij = 1, we will bear the entire cost for increasing the capacity of the arc (i,j), if xij = 0, the arc will not be considered and its capacity remains the same. The objective function is a minimization of the total cost for increasing the capacity of the arcs, ∀(i,j) ∈ A.

min ∑ aij

(i,j)∈A

xij

= min (52 ∗ x12 + 78 ∗ x13 + 182 ∗ x14 + 208 ∗ x26 + 130 ∗ x23 + 52 ∗ x36 + 104 ∗ x35 + 92 ∗ x45 + 26 ∗ x56 + 52 ∗ x58 + 207 ∗ x67 + 52 ∗ x78)

**3.3 CONSTRAINTS**

The flow conservation constraint is the same of the previous problem, except for the value of bi:

∑ xji (j,i)∈BS(i)

− ∑ xij (i,j)∈FS(i)

= bi,∀i ∈ N

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−x12 − x13 − x14 = −1

x12 − x23 − x26 = 0

x13+x23 − x35 − x36 = 0

x14 − x45 = 0

x45 + x35 − x56−x58 = 0

x26 + x36 + x56 − x67 = 0

x67 − x78 = 0

x58 + x78 = 1

The capacity constraint is no more part of the constraints for this problem, in fact now it is included in the objective function thanks to the new data aij. In addition, since the decision variable is binary, we can relax the non-negativity constraint, by imposing xij to be binary.

xij ∈ {0,1},∀(i,j) ∈ A

**4. Solution to point 4**

By running the problem wrote in AMPL language, the solution obtained from the solver is to enlarge the arcs: (1,3), (3,5), (5,8), as we can see from the image below. The total cost for enlarging the capacity, is 234.

*Figure 4: Representation in the network of the optimal solution for point 3*